Exercise 14

If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$.

(a) Find the velocity of the rock after one second.

(b) Find the velocity of the rock when $t = a$.

(c) When will the rock hit the surface?

(d) With what velocity will the rock hit the surface?

Solution

Determine the velocity first.

$$\begin{aligned}
v(t) &= H'(t) \\
&= \lim_{h\to 0} \frac{H(t+h) - H(t)}{h} \\
&= \lim_{h\to 0} \frac{[10(t+h) - 1.86(t+h)^2] - [10t - 1.86t^2]}{h} \\
&= \lim_{h\to 0} \frac{[10t + 10h - 1.86(t^2 + 2th + h^2)] - 10t + 1.86t^2}{h} \\
&= \lim_{h\to 0} \frac{(10t + 10h - 1.86t^2 - 3.72th - 1.86h^2) - 10t + 1.86t^2}{h} \\
&= \lim_{h\to 0} \frac{10h - 3.72th - 1.86h^2}{h} \\
&= \lim_{h\to 0} (10 - 3.72t - 1.86h) \\
&= 10 - 3.72t
\end{aligned}$$

Therefore, the velocity at $t = 1$ is

$$v(2) = 10 - 3.72(1) = 6.28 \ \frac{\text{m}}{\text{s}},$$

and the velocity of the rock at $t = a$ is

$$v(a) = 10 - 3.72(a) = (10 - 3.72a) \ \frac{\text{m}}{\text{s}}.$$

The rock hits the surface at $H = 0$. Solve this equation for t to find the time when this happens.

$$H = 0$$

$$10t - 1.86t^2 = 0$$

$$t(10 - 1.86t) = 0$$

$$t = 0 \quad \text{or} \quad 10 - 1.86t = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{10}{1.86} \approx 5.38 \text{ s}$$

The rock hits the surface at about 5.38 seconds. Plug this time into the velocity function to determine how fast the rock is moving at impact.

$$v\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = -10 \ \frac{\text{m}}{\text{s}}$$